Topology of colors

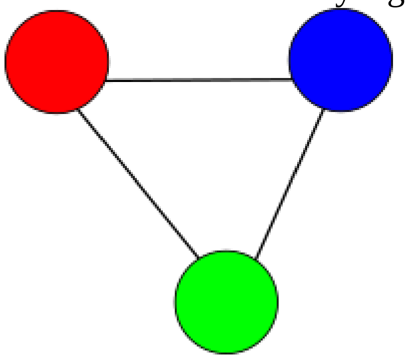
There are two (in some perceived sense) continuous transition from red to blue (over green and purple, respectively) even though these colors are excited by frequencies that are far separated. This fact, I claim, has to do with that we have three color receptors in our eyes. We can imagine a stimulus (normalized to the range 0..1 say) with spectral power (think of it as two lasers with distinct wavelengths corresponding to red and blue) like:

$$(R, G, B) = (\pi/2+\mathrm{atan}(t), 0, \pi/2-\mathrm{atan}(t))/\pi,$$

where t is some parameter going from -∞ to ∞. This corresponds to a fading of color intensity from blue to red. For t=0, these intensities are equal in the red and blue channel. Note that frequency, or wavelength, does not appear explicitly in the expression, but is encoded in the RGB channels. What is expressed is rather the intensity of each frequency.

These colors will now excite the color receptors in our eyes. Thus, there will be some signal transmitted to the brain from the red and the blue color receptors.

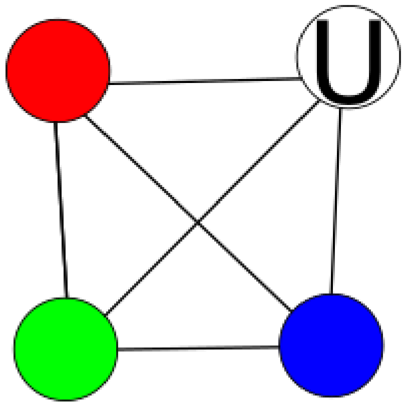
All transitions of the type above can be described by a graph as:



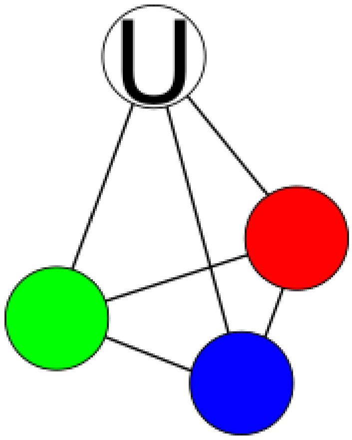
Nodes in the graph correspond to the receptors and edges to the different possible intensity transitions. Note that the graph is fully connected, that is, all nodes have a direct edge to all other nodes. All possible transitions are shown since the graph is fully connected.

All transitions that can exist are represented by the edges in the graph. Since we can write an expression for this type of transition, we can expect it to exist/be possible. Further, all transitions can be on the form above, thus they can be continuous. Since they can be continuous, it is natural to think that the perceived *color* should change continuously too. The graph topology is equivalent to a circle, creating naturally the color circle.

Now here is the twist. Assume instead, that we had 4 color receptors, say the normal ones plus a receptor for UV denoted U. There are now many more transitions that can be formed than the ones above. A fully connected graph represents all of them. Creating the fully connected graph, it looks like:



First, it is clearly not equivalent to a circle any more. This indicate that a being with four frequency receptors will have to have a much more complex representation of the stimuli than color to fully represent the excitation. Remember that all transitions can be made continuous. Hence the perception should also be continuous. Thus, we need to find which topological structure this corresponds to. Rearranging the graph slightly will revile that is can be drawn on a sphere without any edges crossing:



Thus, I predict that a being with four wavelength receptors will represent their stimuli by a 2D vector on a sphere. We could call this *trilor* for this representation, changing the "co" in color to a "tri" from the 3D nature of the representation.

By looking at the topology of these stimulus graphs, one can argue for a particular representation of the stimuli. It is in some sense the simplest representation. The one with the minimal number of parameters associated with it. If another representations was used, that one could be reduced to the minimal one described by the graph topology. This kind of reduction is an interpretation (in the mind) and thus is not the fundamental quantity used in the consciousness.